How Temperature Affects a Servomotor's Electrical and Mechanical Time Constants

Richard H. Welch Jr., Member, IEEE
Welch Enterprise
7819-31st St. N
Oakdale, MN 55128
T: 651-777-6066
F: 651-777-0468
E: welch022@tc.umn.edu

George W. Younkin, Life Fellow, IEEE
Industrial Controls Consulting Division
Bullseye Marketing Inc.
104 South Main St., Suite 320
Fond du Lac, WI 54935
T: 920-929-6544 F: 920-929-9344
E: george@bullseyenet.com

I. INTRODUCTION

In the successful application of an industrial servo system there are two performance requirements. First the servomotor must accurately reproduce all the commanded control variables such as position, velocity, acceleration, jerk along with delivering the required force/torque. Second the servomotor-drive system must remain stable under all applicable operating conditions. To meet both the accuracy and stability requirement the servomotor and its drive electronics must be correctly sized and properly compensated.

Correct motor sizing means the motor has sufficient force/torque output allowing both the motor and its load to accurately reproduce the commanded motion profile. While producing this profile it is also imperative that the motor's maximum continuous or rated operating temperature not be exceeded at any time. In combination with the motor correct drive sizing means that the drive has sufficient output voltage and current allowing the motor to produce the needed force/torque thereby allowing the motor-load to attain its commanded jerk, acceleration, velocity, and position.

Regarding system stability, a system is defined as becoming "unstable" if a disturbance applied to the system produces a response that grows uncontrollably even after this disturbance is removed. In practice servo system stability can be a difficult issue because an improperly compensated motor-drive system can become unstable even though both the motor and its drive are correctly sized [1,2,3]. In analyzing both the motors dynamic motion response and the system stability it is imperative that all the components be accurately identified [1,4]. This identification generally takes the form of a differential equation. As will be shown two key parameters in a motor’s dynamic motion equation are its electrical and mechanical time constants. However, contrary to their implied name, both time constants are not of "constant" value. Rather, both time constants are functions of the motor's temperature. Hence, the purpose of this paper is to discuss the affect temperature has on a motor's electrical and mechanical time constants.

II. DYNAMIC MOTION EQUATION

In dynamic response to an applied voltage command, a servomotor responds both electrically and mechanically. To derive this response the motor’s total voltage equation is combined with its total torque equation. For simplicity a brush type servomotor with cylindrical geometry is used as the model. However, the derived response applies to all servomotors including brushless dc, ac induction, and variable reluctance motors with both linear and cylindrical geometry.

The total voltage equation a permanent magnet brush type dc servomotor with cylindrical geometry amounts to:

$$V(t) = L\frac{di(t)}{dt} + i(t)R(T) + K_E(T)\omega(t). \qquad (1)$$

In equation (1) the parameters are defined as follows:

V(t) = Applied Voltage command (volt)
t = Time (sec).
L = Motor's Inductance (henry)
i(t) = Motor's Current (amp)
d/dt = Time derivative (1/sec)
T = Motor's operating Temperature (ºC)
R(T) = Motor's Resistance (ohm) @ T
K_E(T) = Back EMF (volt/rad/sec) @ T
$\omega(t) =$ Motor's Velocity (rad/sec).

Notice in equation (1) that both the motor's electrical resistance and its back EMF function are shown to depend on temperature. This functional dependence will be described later.

In conjunction with the motor's total voltage equation, the total torque equation for the same motor amounts to*;

$$K_T(T)i(t) = J\frac{d\omega(t)}{dt} + D\omega(t) + F; \qquad (2)$$

Where;

K_T(T) = Torque function (Nm/amp) @ T
J = Moment of Inertia (Kg-m^2)
D = Damping coefficient (Nm/rad/sec)
F = Friction torque (Nm).

{*Note: In applications where a load is attached to the motor the load's inertia, damping, and friction are added to the motor's to obtain the total torque equation for the motor-load system. However, simply adding the load's values to the motors assumes that the shaft or linkage connecting the load to the motor is infinitely stiff. Based on considerable experience this infinite stiffness approximation is not generally valid especially when investigating system stability [1,2,3].}

Based on experience it is reasonable to ignore the motor's damping coefficient when calculating the motor's dynamic motion response. Hence, it is assumed D = 0. With this assumption, solving for the motor's dynamic Current in equation (2) and substituting this expression into equation (1) the equation describing the motor's dynamic motion response to an applied voltage command amounts to:

$$\frac{V(t)}{K_E} = \frac{LJ}{K_E K_T}\frac{d^2\omega(t)}{dt^2} + \frac{RJ}{K_E K_T}\frac{d\omega(t)}{dt} + \omega(t) + \frac{RF}{K_E K_T}. \qquad (3)$$

As defined by NEMA [5]; the motor's electrical time constant is the time required for the Current to reach 63.2% of its final value after a zero source impedance stepped input voltage is applied to a motor maintained in its locked rotor or stalled condition (i.e., $\omega = 0$). Mathematically, the motor's electrical time constant is defined as:

$$\tau_e \equiv \frac{L}{R(T)} \text{ (sec).} \qquad (4)$$

Correspondingly, NEMA defines the motor's mechanical time constant as the time required for an unloaded motor to reach 63.2% of its final velocity after a zero source impedance stepped input voltage is applied to the motor. Mathematically, the motor's mechanical time constant is defined as:

$$\tau_m \equiv \frac{R(T)J}{K_E(T)K_T(T)} \text{ (sec).} \qquad (5)$$

Using these definitions equation (3) can be rewritten as follows:

$$\frac{V(t)}{K_E(T)} = \tau_m\tau_e\frac{d^2\omega(t)}{dt^2} + \tau_m\frac{d\omega(t)}{dt} + \omega(t) + \frac{F}{J}\tau_m. \qquad (6)$$

Notice in equation (6) there are four components in every servomotor's dynamic motion response to the applied voltage command. First there's the motor’s "jerk" multiplied by both the electrical and mechanical time constants. Second there's the motor’s "acceleration" multiplied by the mechanical time constant only. Third there's the motor’s "velocity" which attains a maximum value determined by the applied voltage and the motor’s back EMF function. Finally, there’s motor and applied friction that produces a “drag” on the motor thereby reducing its maximum attainable velocity for a given applied voltage.

To accurately determine how each servomotor dynamically responds to a voltage command equation (6) must solved completely. Although solving equation (6) proves to be instructive [6], it is not the purpose of this paper. The purpose is to determine how the motor's operating temperature affects its electrical and mechanical time constants. In turn equation’s (4) and (5) show that both time constants correspondingly change with temperature. Furthermore, equation (6) shows that as the time constants change with temperature so does every component in the motor’s dynamic motion response.

III. TIME CONSTANTS vs. TEMPERATURE

As shown in equations (4) and (5) any change in the motor's electrical resistance affects both its electrical and mechanical time constants. In every motor one component to its total electrical resistance is the resistance of its primary electrical winding. In all brushless dc, ac induction, and variable reluctance motors the motor’s primary winding is an integral part of its stator. Hence, in these types of motors the total electrical resistance is determined entirely by the stator's winding resistance. In all permanent magnet brush type dc servomotors the electrical winding is contained in the armature for motors with cylindrical geometry and in the actuator or platen in linear motors. In brush type motors, the motor's total electrical resistance is the sum of the actuators or armatures winding resistance plus the added resistance of the brushes, commutator, and the brush-commutator "film". Even though these additional resistive components are part of the motor's total electrical resistance, they typically amount to less than 10% of the total and their temperature dependence, especially for the brush-commutator film, is difficult to predict [7]. Hence, only the change in resistance with temperature of the motor’s primary winding is considered.

The resistance of every electrical winding, at a specified temperature, is determined by the length, gauge, and composition (i.e. copper, aluminum, etc.) of the wire used to construct the winding. The primary winding in the vast majority of industrial servomotors is constructed using film coated copper magnet wire. However, one notable exception is a family of high performance motors known as moving coil motors. The armatures in moving coil motors are often made of aluminum wire and this type of motor is widely used in high acceleration applications such as integrated circuit wire bonding machines, high-speed tape decks, and specific machine tools. However, only copper wire is considered for this discussion.

Based on the 1913 International Electrical Commission standard, the linear temperature coefficient of electrical resistance for annealed copper magnet wire is 0.00393/°C. Therefore, knowing a copper winding's resistance at a specified temperature, the winding’s resistance at temperatures above or below this specified temperature is given by:

$$R(T) = R(T_0)[1 + 0.00393(T - T_0)]; \qquad (7)$$

Where:

T = Winding's Temperature (°C)
T_O = Specified Temperature (°C).

Using equation (7) one learns that a 130°C rise in a copper windings temperature increases its electrical resistance by a factor of 1.5109. Correspondingly, the motor’s mechanical time constant increases by the same 1.5109 factor while its electrical time constant decreases by a factor of 1÷1.5109 = 0.662. In combination, the motor's mechanical to electrical time constant ratio increases by a factor of 2.28 and this change significantly affects how the servomotor dynamically responds to a voltage command [6].

Consulting published motor data one finds that motor manufacturers often specify their motor's parameter values, including resistance, using 25ºC as the specified ambient temperature. NEMA, however, recommends 40°C as the ambient temperature when specifying motors for industrial applications. Therefore, pay close attention to the specified ambient temperature when consulting or comparing published motor data. Different manufacturers can and do, use different ambient temperatures when specifying what can be the identical motor.

In this same published data one also finds that servomotors are generally rated to operate at a maximum continuous winding temperature of 130ºC (Class B) or 155ºC (Class F). Although, motors rated Class H, 180ºC are available. Assuming the motor's electrical resistance, electrical and mechanical time constants are all specified at 25ºC, it was just demonstrated that all three parameter values change with increasing motor temperature. If the motor can operate at 180ºC then the change in resistance is even greater since equation (7) shows a 155ºC rise in winding temperature increases its resistance by a factor of 1.609. Hence, if the servomotor's dynamic motion response is calculated using its 25ºC parameter values then this calculation overestimates the motor's response for all temperatures above 25ºC [6].

In all permanent magnet motors there is an additional affect that temperature has on the motor's mechanical time constant only. As shown in equation (5), the motor's mechanical time constant changes inversely with the motor's temperature dependent back EMF and torque functions, K_E and K_T. In reference [8] it is shown that both K_E and K_T have the same functional dependence on the magnetic flux density produced by the motor's magnets. All permanent magnets are subject to both reversible and irreversible demagnetization [9,10]. Irreversible demagnetization can occur at any temperature and must be avoided by limiting the motor's peak current such that, even for an instant, it does not exceed the peak value specified by the motor manufacturer. Exceeding the motor's peak current value can permanently reduce the motor's K_E and K_T thereby increasing the motor's mechanical time constant at every temperature including the specified ambient temperature.

Reversible thermal demagnetization depends on each specific magnet material. Presently, there are four different magnet materials being used in permanent magnet motors. These four magnet materials are: Aluminum-Nickel-Cobalt(Alnico), Samarium Cobalt(SmCo), Neodymium-Iron-Boron(NdFeB), and Ferrite or Ceramic. Within the temperature range, -60ºC < T < 200ºC, all four magnet materials exhibit linear, reversible thermal demagnetization such that the amount of magnetic flux density produced by each magnet decreases linearly with increasing magnet temperature. Hence, similar to electrical resistance, the expression for the, reversible linear decrease in both $K_E(T)$ and $K_T(T)$ with increasing magnet temperature amounts to:

$$K_{E,T}(T) = K_{E,T}(T_0)\left[1 - B(T - T_0)\right]. \qquad (8)$$

In equation (8), the B-coefficient for each magnet material amounts to:

B (Alnico) = 0.0001/ºC
B (SmCo) = 0.00035/ºC
B (NdFeB) = 0.001/ºC
B (Ferrite) = 0.002/ºC.

Using equation (8), one calculates that a 100ºC rise in magnet temperature causes a reversible decrease in the motor's K_E and K_T that amounts to 1% for Alnico, 3.5% for SmCo, 10% for

NdFeB, and 20% for Ferrite or Ceramic magnets.

Like the motor's electrical resistance, most motor manufactures specify the motor's K_E and K_T using the same ambient temperature to specify resistance. However, this is not always true and it is again advised to pay close attention how the manufacturer is specifying their motor's parameter values.

Combining the affect of reversible, thermal demagnetization with temperature dependant electrical resistance, the equation describing how a permanent magnet motor's mechanical time constant increases in value with increasing motor temperature amounts to:

$$\tau_m(T) = \tau_m(T_0)\left[\frac{1+0.00393(T-T_0)}{\left(1-B(T-T_0)\right)^2}\right]. \qquad (9)$$

Notice in equation (9) that the magnet temperature is assumed equal to the winding temperature. Actual measurement shows this assumption is not always correct. The motor’s magnets often operate with a lower temperature compared to its winding temperature. However, this conservative approximation is recommended.

Figure 1 shows four graphs of the mechanical time constant multiplier (i.e., $\tau_m(T)/\tau_m(T_0)$) as a function of temperature for four different motor conditions. Notice the specified ambient temperature is 25ºC. The R-only graph is the mechanical time constant multiplier due to a change in electrical resistance only that occurs in every servomotor. The remaining three graphs show the combined affect of increasing electrical resistance with thermal demagnetization for three different magnet materials. As shown, the largest change occurs in Ferrite motors with the mechanical time constant increasing by a factor of 2.759 at 155ºC. The second largest change occurs in NdFeB motors as the multiplier is 1.99 at 155ºC. Finally, SmCo motors show less change than either Ferrite or NdFeB motors. The Alnico graph is not shown because it is difficult to distinguish from the R-only graph.

IV. RECOMMENDATION

This paper shows that a Servomotor's dynamic motion response is controlled by both its electrical and mechanical time constants. As shown, both time constants depend on temperature. As a minimum, the increase in every motor’s electrical resistance with increasing winding temperature causes the motor’s mechanical time constant to increase in value while its electrical time constant decreases in value. In all permanent magnet motors, including the widely used brushless dc, there’s the added affect of reversible thermal demagnetization that reduces the motor’s K_E and K_T thereby further increasing the mechanical time constant only. Ignoring these time constant changes can result in a significant error when calculating the motor's dynamic motion response [6]. In turn, this error can result in the wrong motor being selected for the application thereby leading to costly production delays and unhappy customers. To avoid making this mistake, it is recommended that the motor’s dynamic motion response be calculated using the motor’s electrical and mechanical time constant values adjusted to the motor’s maximum continuous or rated operating temperature.

V. REFERENCES

[1] R. Welch Jr., “The Effects of Mechanical Resonance in a Closed Loop Servo-system; Problems and Solutions”, A Tutorial Available from Welch Enterprise.

[2] G. Younkin, W. McGlasson, and R. Lorentz, “Considerations for low inertia AC drives in machine tool servo applications”, IEEE Transactions on Industry Applications, Vol. 27, No. 2, March/April 1991.

[3] G. Ellis and R. Lorentz, "Resonant load control methods for industrial servo drives", IEEE Industry Application Society Annual Meeting Rome, Italy, October 8-12, 2000.

[4] B.P. Lahti, "*Signals, Systems and Communications*", Copyright © 1965 by John Wiley & Sons, Inc.

[5] NEMA-National Electric Manufacturers Association, "*Motion/Position Control Motor, Controls, and Feedback Devices*", MG 7-1989, Section 1, Motors.

[6] R. Welch Jr., "Optimizing a Motor's Dynamic Motion Response" A Tutorial available from Welch Enterprise.

[7] Morganite, "*Carbon Brushes and Electrical Machines*", Copyright © 1978, Available from Morganite Carbon Ltd.

[8] R. Welch Jr., "The Basics of Permanent Magnet Motor Operation", A Tutorial Available from Welch Enterprise.

[9] R. M. Bozorth, "*Ferromagnetism*", Copyright © 1951, D. Van Nostrand Co. Inc., Reprinted 1993 by IEEE press.

[10] R. Welch Jr., " Demagnetization can cause reversible and irreversible changes in permanent magnet motor performance", Power Conversion and Intelligent Motion (**PCIM**), July 1995, pp. 35-44.

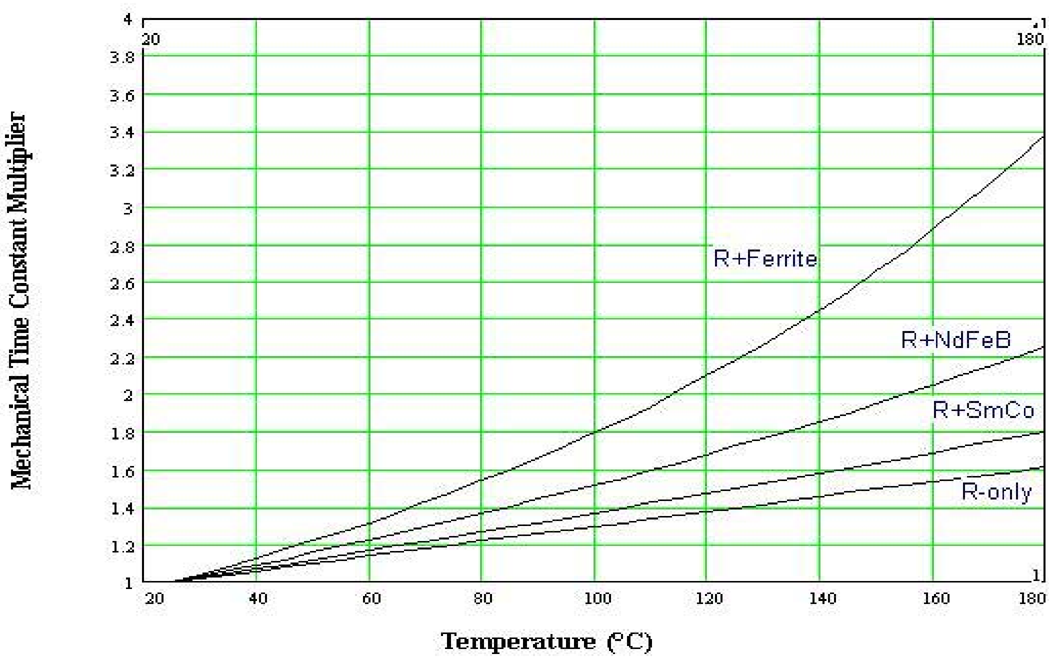

Figure 1. Mechanical Time Constant Multiplier versus Temperature