RAMIFICATIONS of POSITION SERVO LOOP COMPENSATION

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For many years industrial positioning servo drives did not use servo compensation in the forward position loop. This was referred to as a "naked" servo. The reason was that integral compensation in the forward position loop could result in undesirable overshoots. This was especially evident in machine-tool contouring operations where a corner was required in the machined part. For these operations, any overshoot in a positioning servo would remove undesired metal on an inside corner in the machined part.

To study these overshoot phenomena a series of transient response analyses are made for typical industrial positioning servos. It is assumed that the internal velocity servo has a 30 Hz (188 rad/sec) bandwidth. Further, to simplify the analyses, it will be assumed that the load dynamics have a mechanical resonance 3 times the velocity servo bandwidth (30 Hz), and performance will not be degraded sufficiently to affect the transient analysis. Further it will be assumed that the servo drive has been sized properly to provide sufficient torque to overcome friction loses, accelerate the load properly, and provide sufficient machine thrust as needed.

To proceed with these analyses, it is necessary to write the complete closed loop differential equations with initial conditions, and then solve for the time response equation for the following responses:

A Velocity acceleration to a final value.
B Position acceleration response.
C Velocity deceleration from an initial velocity.
D Position deceleration from an initial position.

NAKED POSITION SERVO

As a control, the first model to be studied will be for a classical "naked" position servo drive with a velocity constant (K_v) = 1 ipm/mil= 16.66/sec. The minor velocity servo loop has a typical bandwidth of 30 Hz. The Bode plot for this model is shown if figure 1, and the mathematical proof is given in appendix I. The four transient response tests are-

A The velocity acceleration response is shown in figure 2 for a step input to 100 ipm. The response is normal with no overshoot in velocity.

B The position acceleration of figure 3, is a linear ramp as expected.

C The velocity deceleration of figure 4 is as expected from an initial constant velocity of 10 ipm with no overshoot.

D The position deceleration from an initial position of 10 inches is shown in figure 5 with no overshoot.

10/1 LAG-LEAD POSITION SERVO

This model uses a lag/lead compensation in the forward position loop. The apparent velocity constant (K_v) = 158/sec. An internal velocity servo loop exists with a 30 Hz bandwidth. The Bode plot for this model is shown in figure 6, with the mathematical proof given in appendix II. The four transient response tests are-

A The velocity acceleration response to a 100 ipm step in velocity is shown in figure 7. A small overshoot in velocity is shown.

B The position response to a 100 ipm step in velocity is shown in figure 8. The response is unremarkable.

C The velocity deceleration response from an initial constant velocity of 10 ipm is shown in figure 9 with a small overshoot as expected with integral compensation in the position forward loop.

D The position deceleration from an initial position of 10 inches is shown in figure 10. The overshoot in the position response is undesirable for industrial machine positioning servos being used in a contouring mode of operation.

TYPE 2 POSITION SERVO

The type 2 position loop compensation uses proportional plus integral (PI) compensation in the forward position loop. The position loop gain was chosen as 79/sec (4.7 ipm/mil). The internal velocity servo has a bandwidth of 30 Hz. A Bode plot for this model is shown in figure 11 with the mathematical proof given in appendix III. The transient response tests are-

A The velocity acceleration response to a 100 ipm step in velocity is shown in figure 12. The amount of overshoot above 100 ipm is about 116 ipm or about 16 percent. This amount of overshoot is larger than the comparable 10/1 lag-lead compensation model of figure 7.

B The position response to a 100 ipm step in velocity is shown in figure 13 and is unremarkable.

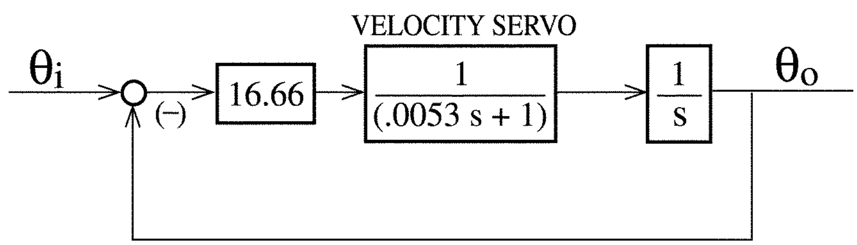
C The velocity deceleration response from an initial constant velocity of 10 ipm is shown in figure 14 with an overshoot in the forward position loop. The amount of overshoot is about twice as much in comparison to the 10/1 lag-lead compensation of figure 9.

D The position deceleration response from an initial position of 10 inches is shown in figure 15. The overshoot is about twice as great as the 10/1 lag-lead compensation model of figure 10. It is undesirable for industrial machine positioning servos being used in a contouring mode of operation.

DISCUSSION

For industrial positioning servo drives a "naked" compensation (no integral compensation in the forward position loop) does not indicate any overshoot in the deceleration mode for velocity or position. For machining operations on industrial machines this is an important requirement in such machining operations into square corners, etc.

For industrial positioning servo drives using lag/lead compensation or proportional plus integral (PI) compensation, and type 2 control, the servo drive will exhibit overshoot characteristics during deceleration. The larger the initial velocity, the greater the overshoot during deceleration. Of the two types of position loop compensation, the type 2 servo exhibits greater overshoot.



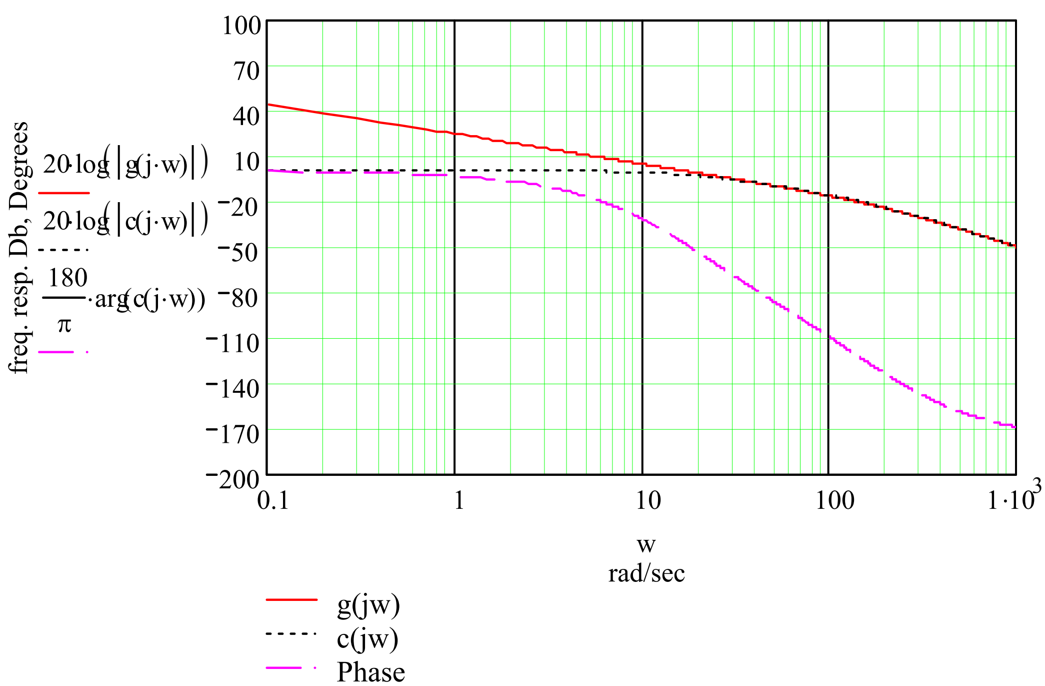


Fig 1 Naked Pos. Servo Rersponse

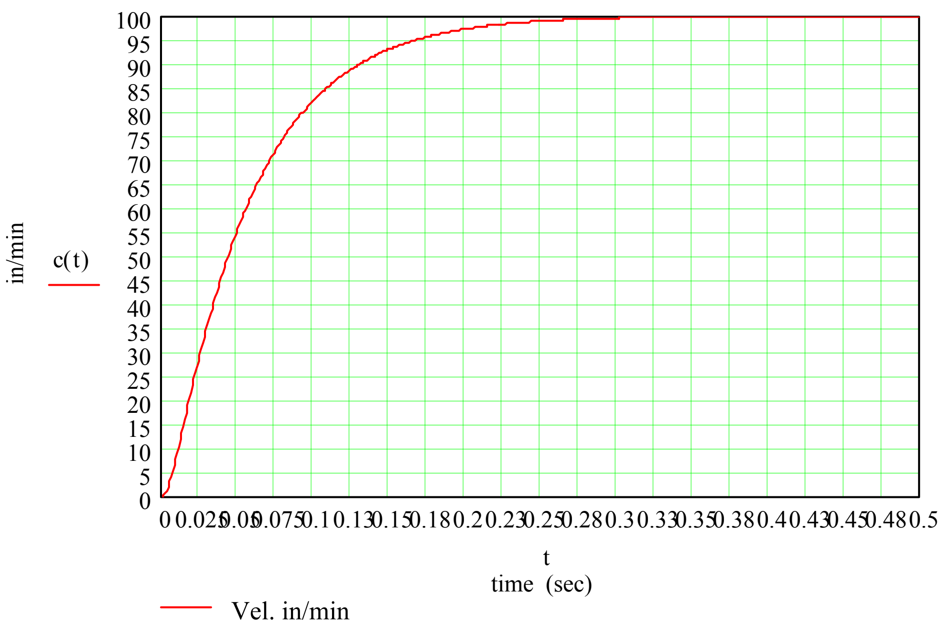


Fig 2 Vel. Accel.-Naked servo

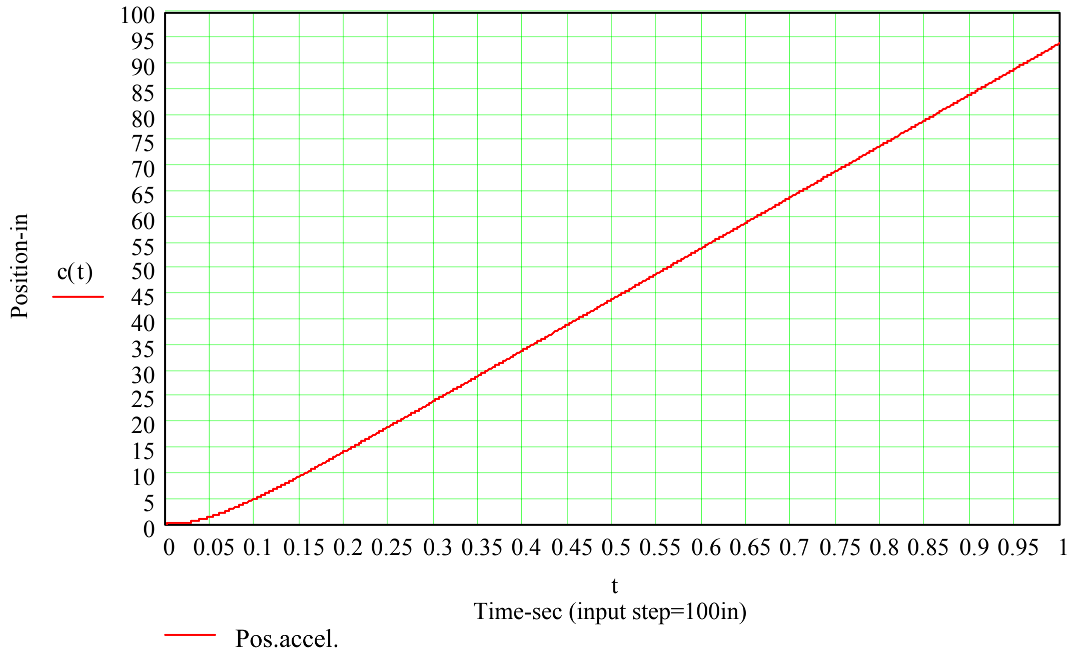


Fig. 3 Pos. Accel.-Naked servo

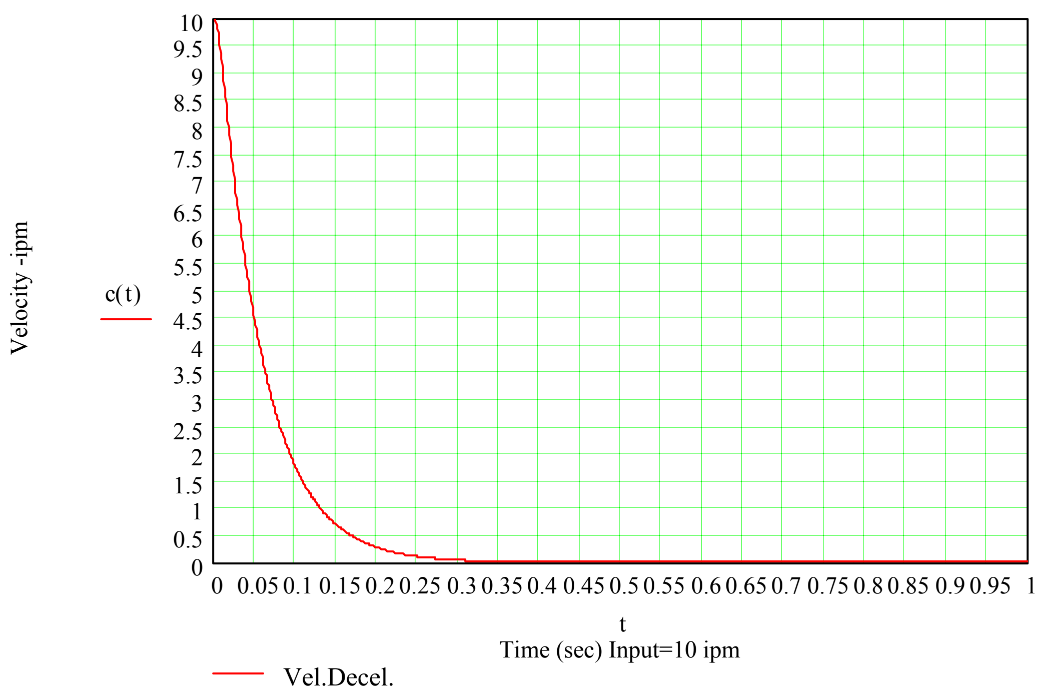


Fig. 4 Vel. Decel.-Naked servo

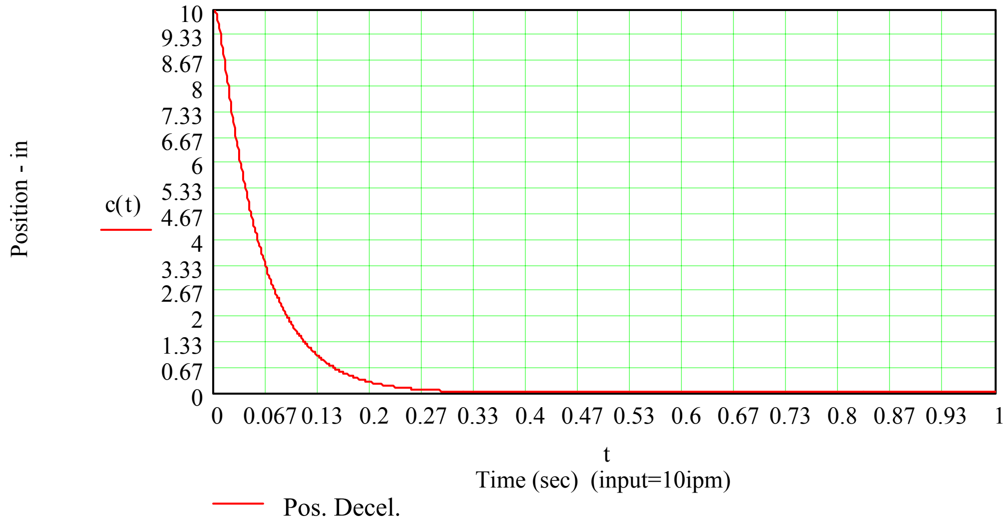
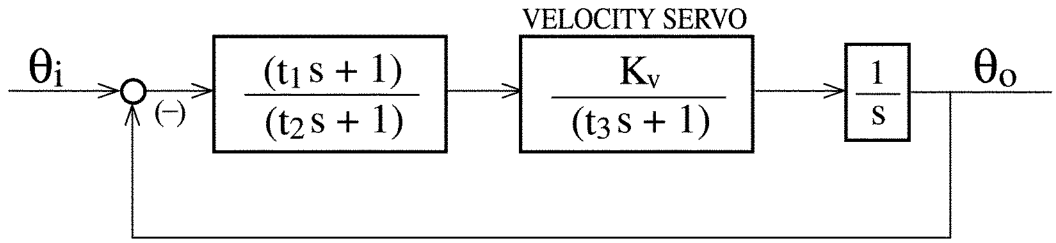


Fig. 5 Position Decel. -Naked servo



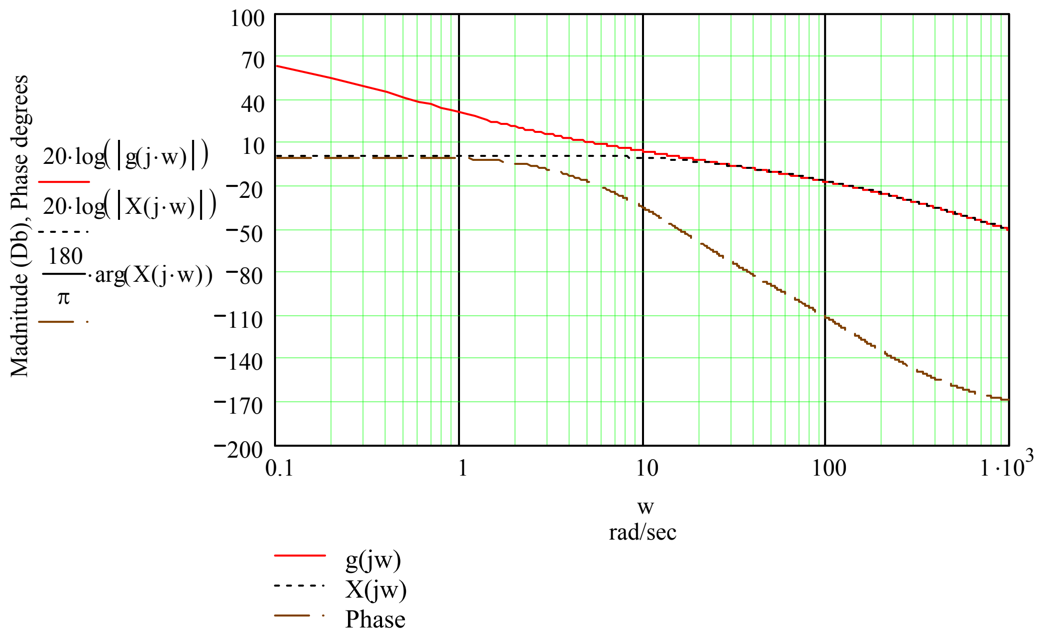


Fig. 6 Lag/lead pos. comp.

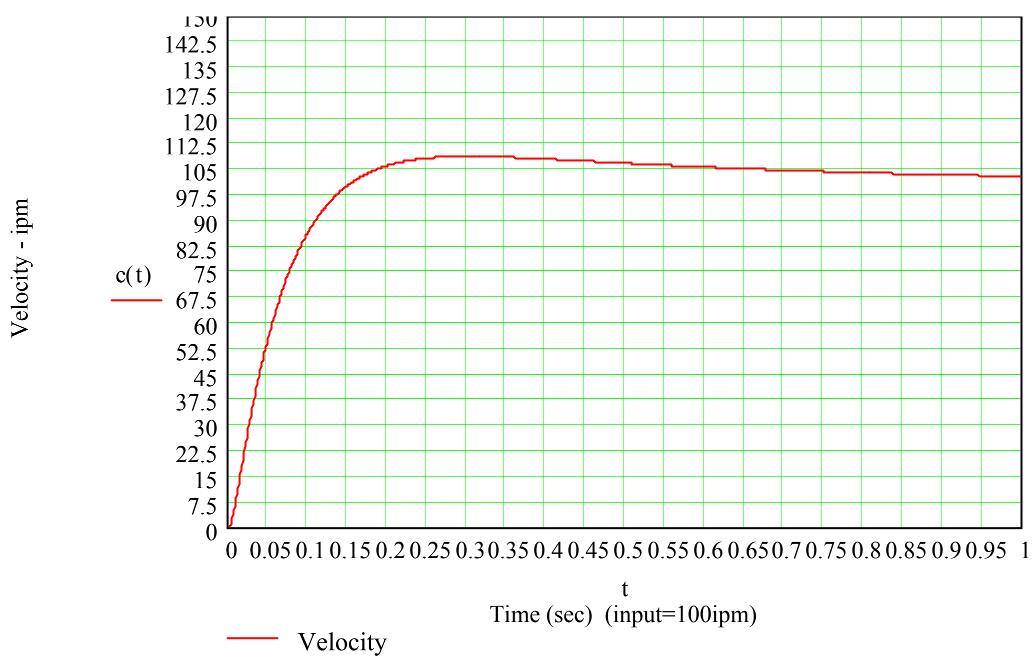


Fig. 7 Vel. Accel.-10/1 lag-lead

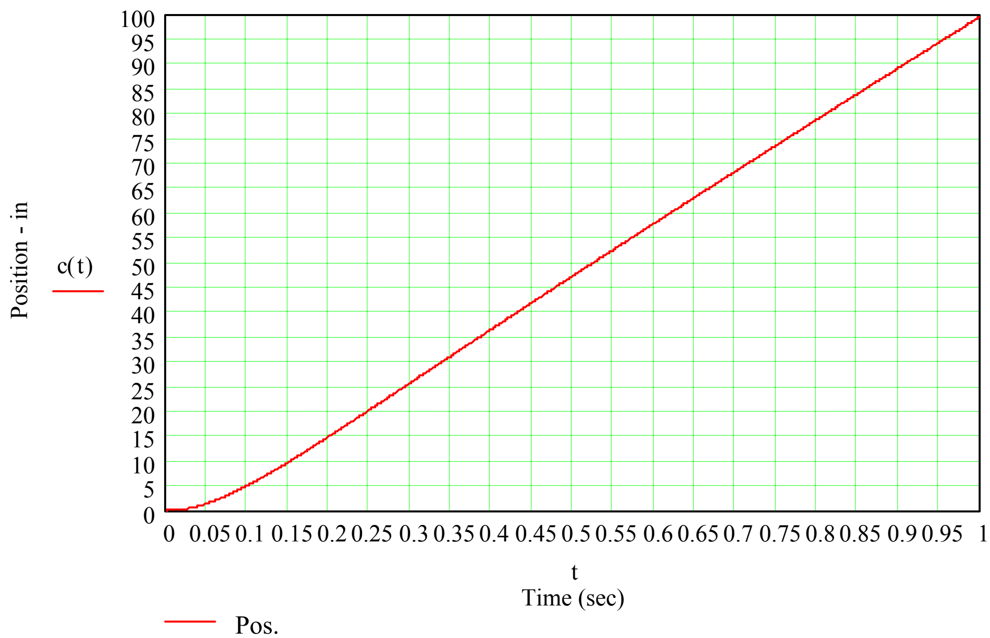


Fig. 8 Pos. Accel. (10 to 1 lag-lead)

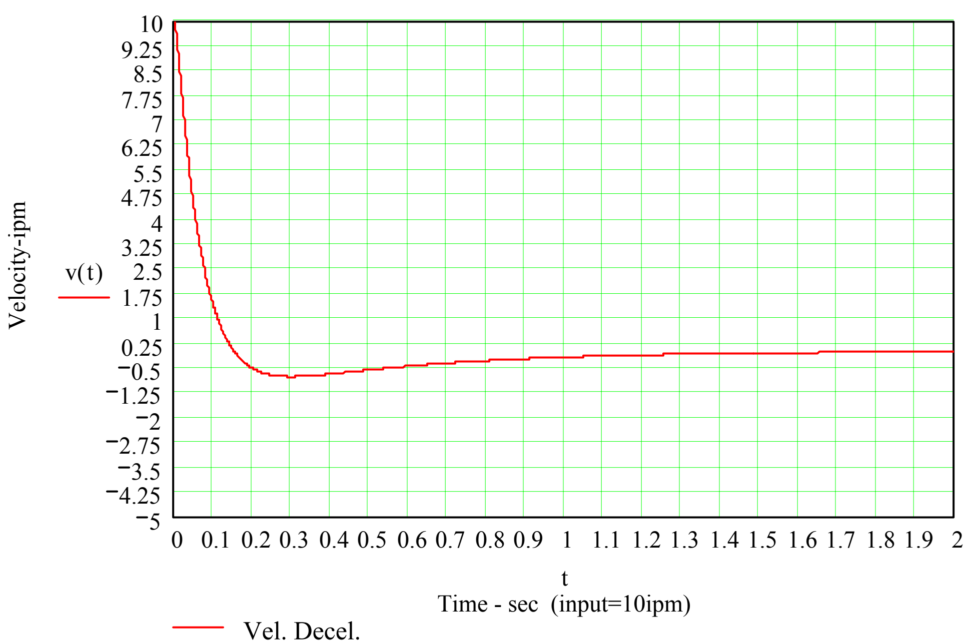


Fig. 9 Velocity Decel.(10 to 1 lag-lead)

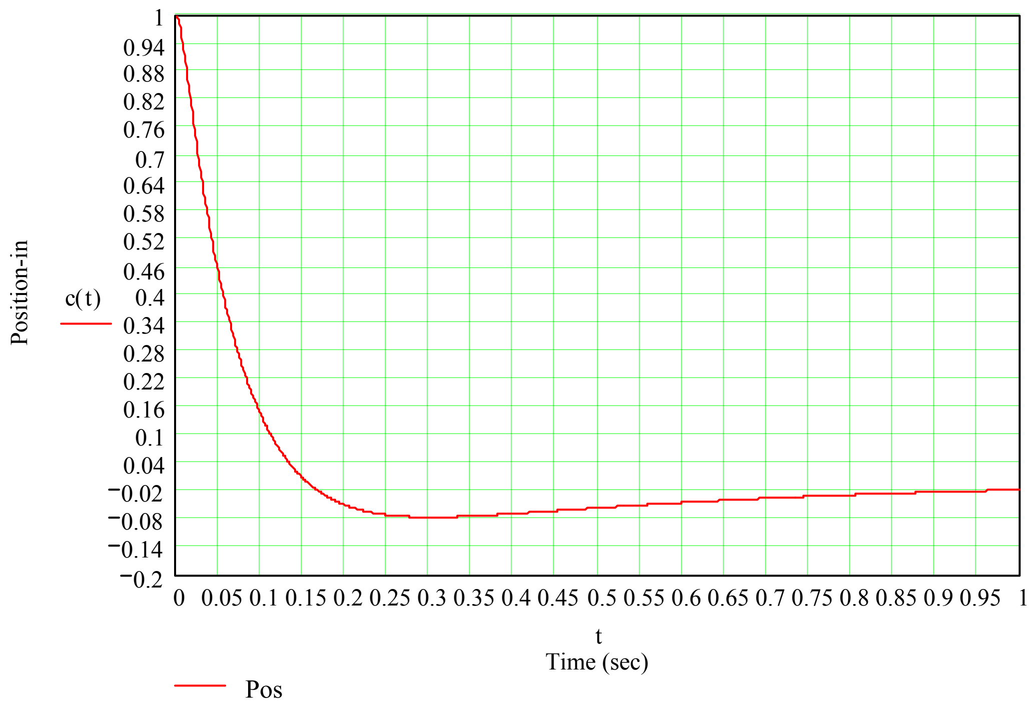
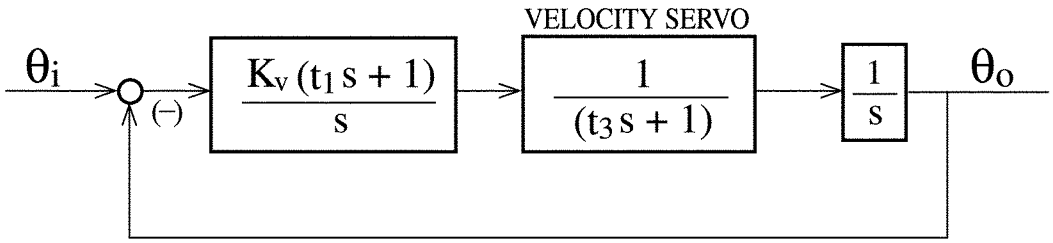


Fig. 10 Pos. decel.(10 to 1 lag-lead)



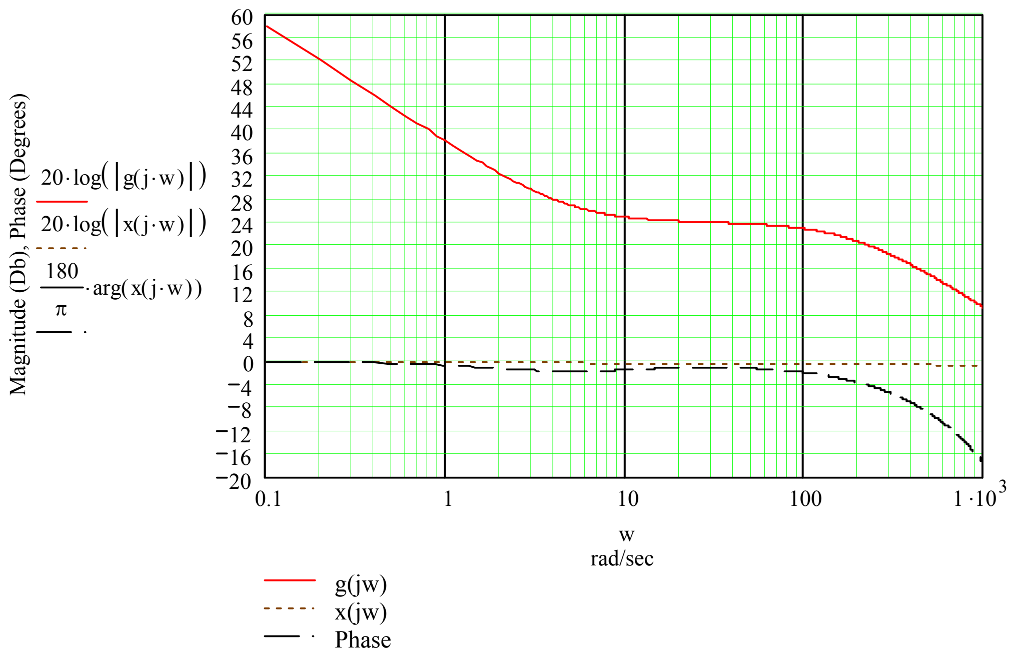


Fig. 11 Type 2 Servo Freq. Response

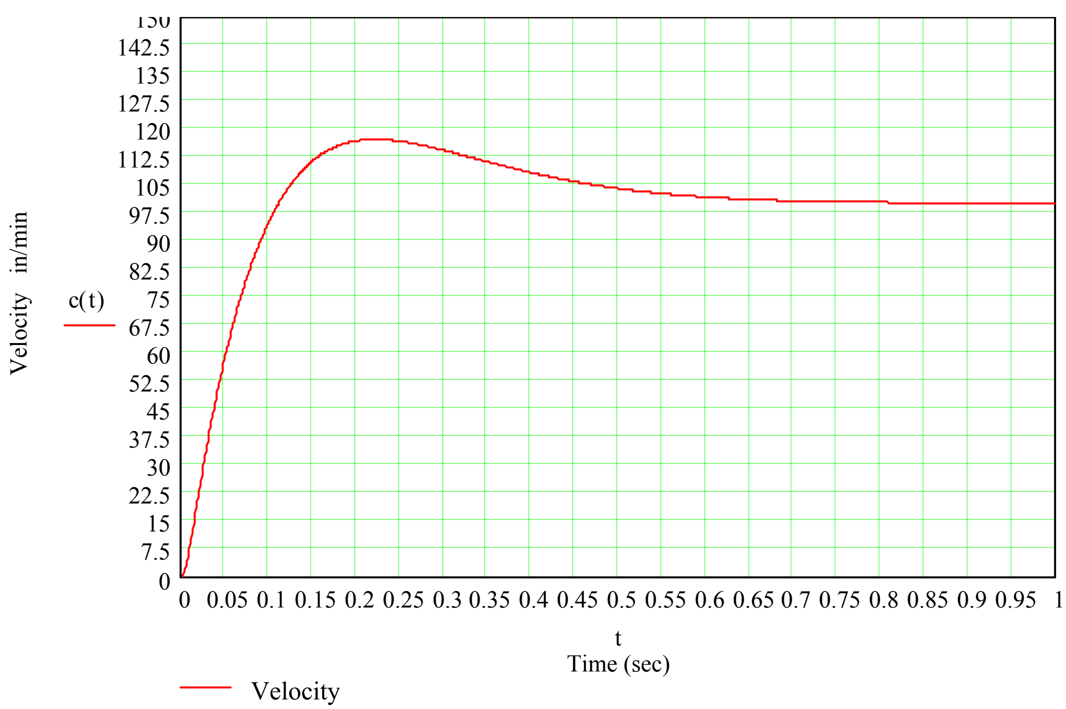


Fig. 12 Velocity Accel. Type 2 Servo

Fig. 13 Position Accel. (Type 2 Servo)

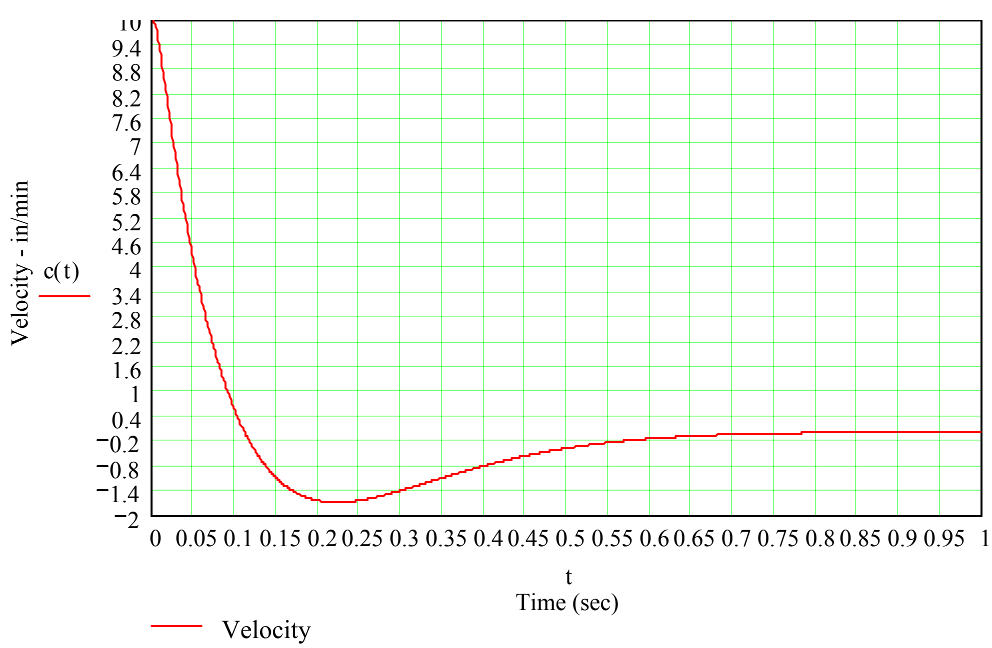


Fig. 14 Velocity Decel. (Type 2 Servo)

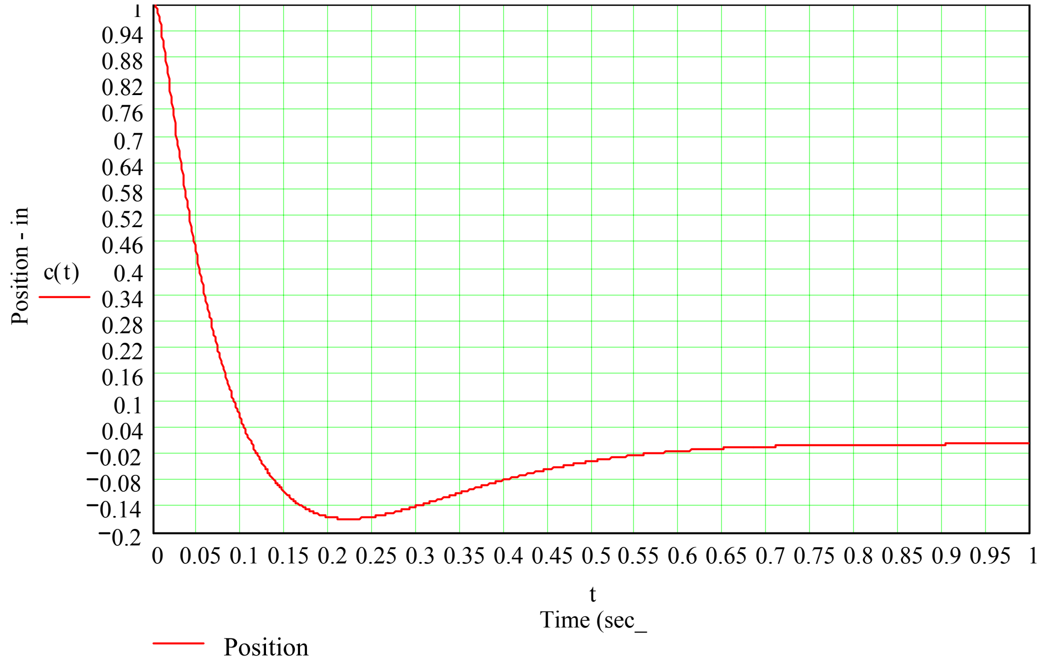
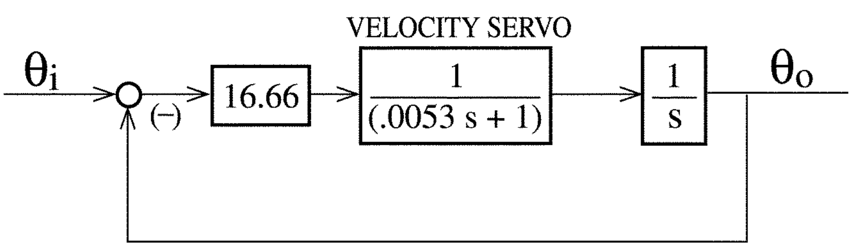


Fig. 15 Position Decel (Tyep 2 Servo)

APPENDIX I

"NAKED SERVO COMPENSATION"



$$\frac{s\theta_o}{s\theta_i} = \frac{V_o}{V_i} = \frac{16.66}{0.0053s^2 + s + 16.66}$$

$$V_{o(s)} = [0.0053s^2 + s + 16.66] = V_i 16.66$$

$$0.0053(V_o s^2 - sV_o(0) - \dot{V}_o(0)) + (V_o(0)) + V_o\ 16.66 = V_i\ 16.66$$

$$0.0053V_o s^2 - 0.0053V_o(0) - 0.0053\dot{V}_o(0) + V_o s - V_o(0) + V_o\ 16.66 = V_i\ 16.66$$

$$V_o(0) = V_{in} \qquad \dot{V}_o(0) = 0$$

$$V_o(0.0053s^2 + s + 16.66) = 16.66V_i + 0.0053V_{in}s + V_{in}$$

$$V_o(s) = \frac{V_i \quad 16.66}{0.0053s^2 + s + 16.66} + \frac{V_{in}(0.0053s + 1)}{0.0053s^2 + s + 16.66}$$

$$V_o(s) = \frac{V_i \ 3143}{s^2 + 188s + 3143} + \frac{V_{in} \ (s + 188)}{s^2 + 188s + 3143}$$

VELOCITY ACCELERATION

$$V_i = \frac{V_F}{s}$$

$$V_o(s) = \frac{V_F \quad 3143}{s(s^2 + 188s + 3143}$$

Ia

VELOCITY DECELERATION

$$V_o(s) = \frac{V_{in} \quad (s + 188)}{(s^2 + 188s + 3143)}$$

POSITION ACCELERATION

$$X_o(s) = \frac{V_F \ 3143}{s^2(s^2 + 188s + 3145)}$$

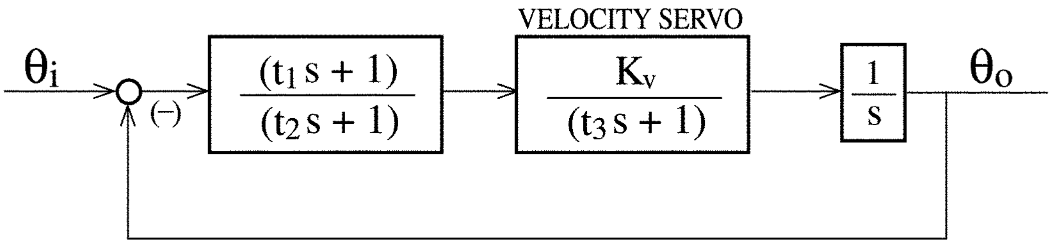
POSITION DECELERATION

$$X_o(s) = \frac{X_{in} \ (s + 188)}{(s^2 + 188s + 3143)}$$

Ib

APPENDIX II

10/1 LAG-LEAD COMPENSATION



$$\frac{X_o}{X_i} = \frac{V_v\ (t_1 s + 1)}{s(t_2 s + 1)(t_3 s + 1) + K_v (t_1 s + 1)}$$

$$sX_o(s(t_2 s + 1)(t_3 s + 1) + K_v(t_1 s + 1)) = sX_i K_v (t_1 s + 1)$$

$$V_o(t_2 t_3 s^3 + (t_2 + t_3)s^2 + (1 + K_v t_1)s + K_v) = V_i K_v (t_1 s + 1)$$

INITIAL CONDITIONS

$$t_2 t_3 (V_o s^3 - s^2 V_o(0) - s\dot{V}_o(0) - \ddot{V}_o(0)) + (t_2 + t_3)(V_o s^2 - sV_o(0) - \dot{V}_o(0)) +$$
$$(1 + K_v t_1)(V_o s - V_o(0)) + K_v V_o = K_v t_1 (V_i s - V_i(0)) + V_i K_v$$

$$V_o(0) = V_{in} \qquad \dot{V}_o(0) = 0 \qquad \ddot{V}_o(0) = 0 \qquad V_i(0) = V_{in}$$

$$t_2 t_3 (V_o s^3 - s^2 V_{in}) + (t_2 + t_3)(V_o s^2 - sV_{in}) + (1 + K_v t_1)(V_o s - V_o(0)) + K_v V_o = K_v t_1 (V_i s - V_{in}) + V_i K_v$$

$$V_o(t_2 t_3 s^3 + (t_2 + t_3)s^2 + (1 + K_v t_1)s + k_v) =$$

$$V_i K_v (t_1 s + 1) - K_v t_1 V_{in} + t_2 t_3 V_{in} s^2 + (t_2 + t_3)sV_{in} + (1 + K_v t_1)V_{in}$$

$$V_o(t_2 t_3 s^3 + (t_2 + t_3)s^2 + (1 + K_v t_1)s + K_v) =$$

$$V_i K_v (t_1 s + 1) - K_v t_1 V_{in} + t_2 t_3 V_{in} s^2 + (t_2 + t_3)sV_{in} + V_{in} + K_v t_1 V_{in}$$

Iia

$$V_o(s) = \frac{V_i K_v (t_1 s + 1)}{t_2 t_3 s^3 + (t_2 + t_3)s^2 + (1 + K_v t_1)s + K_v} + \frac{V_{in}(t_2 t_3 s^2 + (t_2 + t_3)s + 1)}{t_2 t_3 s^3 + (t_2 + t_3)s^2 + (1 + K_v t_1)s + K_v}$$

$K_v = 158/\sec$ $\qquad t_2 t_3 = 0.0265$

$t_1 = 0.5\sec$ $\qquad t_2 + t_3 = 5$

$t_2 = 5\sec$ $\qquad (1 + K_v t_1) = 80$

$t_3 = 0.0053\sec$ $\qquad V_i = \frac{V_F}{s}$

VELOCITY ACCELERATION

$$V_o(s) = \frac{V_F\ 158\ (0.5s + 1)}{s(0.0265s^3 + 5s^2 + 80s + 158}$$

$$V_o(s) = \frac{V_F\ 2981\ (s + 2)}{s(s^3 + 188s^2 + 3018s + 5692)}$$

$$V_o(s) = \frac{V_F\ 2981\ (s + 2)}{s(s + 2.3)(s + 15)(s + 171)}$$

VELOCITY DECELERATION

$$V_o(s) = \frac{V_{in}(0.0265s^2 + 5s + 1)}{0.0265s^3 + 5s^2 + 80s + 158}$$

$$V_o(s) = \frac{V_{in}\ (s^2 + 188s + 37.7)}{(s + 2.3)(s + 15)(s + 171)}$$

POSITION ACCELERATION

$$X_o(s) = \frac{V_F\ 2981(s + 2)}{s^2(s + 2.3)(s + 15)(s + 171)}$$

POSITION DECELERATION

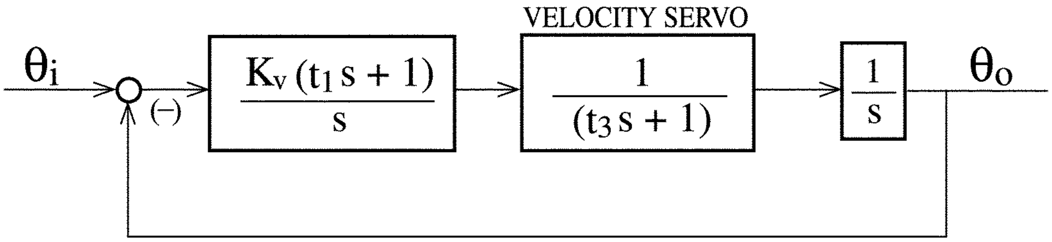
$$X_o(s) = \frac{V_{in}(s^2 + 188s + 37.7)}{s(s + 2.3)(s + 15)(s + 171)}$$

IIb

$$X_o(s) = \frac{X_{in}(s^2 + 188s + 37.7)}{(s+2.3)(s+15)(s+171)}$$

APPENDIX III

TYPE 2 SERVO



$$\frac{\theta_o}{\theta_i} = \frac{K_v(t_1 s + 1)}{s^2(t_3 s + 1) + K_v(t_1 s + 1)}$$

$$\theta_o = (s^2(t_3 s + 1) + K_v(t_1 s + 1)) = \theta_i \; K_v(t_1 s + 1)$$

$$s\theta = V$$

$$V_o(t_3 s^3 + s^2 + K_v t_1 s + K_v) = V_i K_v(t_1 s + 1)$$

Initial conditions

$$t_3(V_o s^3 - s^2 V_o(0) - s\dot{V}_o(0) - \dot{V}_o(\dot{0})) + (V_o s^2 - sV_o(0) - \dot{V}_o(\dot{0})) +$$

$$K_v t_1(V_o s - V_o(0)) + V_o K_v = K_v t_1(V_i s - V_i(0)) + V_i K_v$$

$$V_o(0) = V_{in} \quad , V_o(\dot{0}) = 0 \,, \quad \dot{V}_o(\dot{0}) = 0, \quad V_i(0) = V_{in}$$

$$V_o t_3 s^3 - t_3 s^2 V_{in} + V_o s^2 - sV_{in} + K_v t_1 V_o s - K_v t_1 V_{in} + V_o K_v = V_i(K_v t_1 s + K_v) - K_v t_1 V_{in}$$

$$V_o(t_3 s^3 + s^2 + K_v t_1 s + K_v) = V_i(K_v t_1 s + K_v) - K_v t_1 V_{in} + t_3 s^2 V_{in} + sV_{in} + K_v t_1 V_{in}$$

$$V_o = \frac{V_i K_v(t_1 s + 1)}{t_3 s^3 + s^2 + K_v t_1 s + K_v} + \frac{V_{in}(t_3 s^2 + s)}{t_3 s^3 + s^2 + K_v t_1 s + K_v}$$

IIIa

$$K_v = \frac{79}{\sec} \qquad t_1 = 0.2\sec \qquad t_3 = 0.0053\sec$$

$$V_o(s) = \frac{V_i(15.8s+79)}{0.0053s^3 + s^2 + 15.8s + 79} + \frac{V_{in}(0.0053s^2 + s)}{0.0053s^3 + s^2 + 15.8s + 79}$$

$$V_o(s) = \frac{V_i 2981(s+5)}{s^3 + 188s^2 + 2981s + 14905} + \frac{V_{in}s(s+188)}{s^3 + 188s^2 + 2981s + 14905}$$

$$V_o(s) = \frac{V_i 2981(s+5)}{(s+171)(s+(8.45+j3.94))(s+(8.45-j3.94))} + \frac{V_{in}s(s+188)}{''}$$

$$V_o\,(s) = \frac{V_i 2981(s+5)}{(s+171)(s^2+16.914s+87.12)} + \frac{V_{in}s(s+188)}{(s+171)(s^2+16.9s+87.12)}$$

VELOCITY ACCELERATION

$$V \; = \frac{V_F}{s}$$

$$V_o(s) = \frac{V_F 2981(s+5)}{s(s+171)(s^2+16.9s+87.1)}$$

VELOCITY DECELERATION

$$V_o(s) = \frac{V_{in}s(s+188)}{(s+171)(s^2+1`6.9s+87.1)}$$

POSITION ACCELERATION

$$X_o(s) = \frac{V_F 2981(s+5)}{s^2(s+171)(s^2+16.9s+87.12)}$$

POSITION DECELERATION

$$X_{in} = \frac{V_{in}}{s}$$

$$X_o(s) = \frac{X_{in}s(s+199)}{(s+171)(s^2+16.9s+87.12)}$$

IIIb